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Model of friction to take into account the sliding distance dependence and its memory effect

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ABSTRACT

Materials sliding on a larger substrate age faster than the substrate itself, but no available model of friction takes into account this aspect. We developed a model based on the memory effect of both the mobile and the plane samples, separately, and characterized by macroscopic geometric measurements (e.g., sliding distances and samples sizes). We studied the contact between fresh paper samples and samples that have already underwent several slidings. Experimental and theoretical results are in good agreement. Finally, we used our model to demonstrate how the existing test methods can lead to differences in the measured forces of friction. The results allow both a better understanding and quantitative characterization of the dependence of friction with the history of the contact.

1. Introduction

The force of friction between two surfaces may depend on the distance of relative sliding. This dependence was intensively studied for sliding distances at a microscale (e.g., static-to-kinetic transition [1]) or for long runs (e.g., wear). However, the dependence of the friction force to sliding distances of the scale of the samples (e.g., few centimeters) remains poorly characterized. Yet this dependence influences the stability of stacks [2] or the touch-feel of tissues [3], for example.

To study this influence of the sliding distance on the friction force, we consider the force of kinetic friction induced by a paper-on-paper contact [4]. Indeed, macroscopic sliding distances between papers induce drops in friction force: a logarithmic-shape decrease up to −50% in 30 cm is typically observed [5–8]. Moreover, this decrease has a persistent memory effect [6]: if the experience is interrupted, the next value of friction is equal to the one prior to the stop, even for hours. Finally, the decrease in friction force with the sliding distance is identical considering (i) one direction, (ii) the opposite direction, and (iii) when switching from one direction to the opposite direction [6–8]. A reorientation of surface structures (e.g., cellulosic fibers) in the direction of the sliding may explain these phenomena [7,8].

A model of the friction force evolution is needed to engineer mechanisms involving friction. Such a model is however still missing for sliding distances of the scale of the samples length. Moreover, the friction force is considered in standards (i) during the first and third repeated slidings, (ii) on a distance of 10 cm, (iii) in the same sliding direction, and (iv) from the same starting point, as the solid is repositioned to its original place after each sliding [9–11]. Two measurements are however not sufficient to properly describe the whole decrease, in particular during the first sliding. In addition, the measurements are time consuming, as numerous repeated slidings are required. The decrease in friction force with the sliding distance may be also wrongly assimilated to the transition from the break-away force to the force of kinetic friction: both decreases have the same shape, but (i) take place on different scales, (ii) are due to different mechanisms, and (iii) have different memory effects. Finally, the force of friction measured on a given sliding distance cannot be considered equal to the one obtained for another sliding distance. Thus, comparing the results obtained with two standards using different sliding distances [7] becomes unfounded. Similarly, the force of friction measured after a 10 cm slide cannot be compared to the friction measured in industrial processes, where the typical characteristic length is of few meters.

Characterizing the decrease in friction force with the sliding distance while minimizing the number of experiments is a challenge we propose to face. We will first propose a theoretical model of the decreasing friction force. Then, we will present the materials and method used for both identification and validation of the proposed model. Finally, the

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results of the experiments will be presented, and both the proposed model and the experimental method will be discussed.

2. Proposed friction model

We develop a model that links the force of friction to dimensional parameters – in particular the sliding distance and the sizes of the samples. We consider a *mobile* sample sliding on a *plane* sample. The surface of both samples has surface structures, reoriented in the direction of the sliding. The size of these structures is infinitesimal compared to the size of the samples.

We call $f_{f,0}$ the contribution to the force of friction, per unit apparent contact area, for surface structures that are fully reoriented in the direction of the sliding (expressed in N m^{-2}). When the surface structures are not fully reoriented in the direction of the sliding, the force of friction increases. The contributions of mobile and plane surface structures to this increase, per unit apparent contact area, are noted $f_{f,mobile}$ and $f_{f,plane}$, respectively (expressed in N m^{-2}). Thus, the force of friction, F_f, is calculated as the sum of those different contributions on the whole apparent contact surface, S:

$$F_f = \int_S \left(f_{f,0} + f_{f,mobile} + f_{f,plane} \right) \mathrm{d}S \quad (1)$$

We consider that the mobile and plane surfaces are made from the same material. Their surface structures are therefore of the same nature. Thus, the contribution of these structures to the friction force ($f_{f,mobile}$ and $f_{f,plane}$) can be described by the same function f_f. Literature suggests that the sliding modifies the force of friction. Therefore, we propose that the f_f function depends on the total sliding distances underwent by the surface structures, called *local sliding distances*:

$$\begin{cases} f_{f,mobile} = f_f(d_{mobile}) \\ f_{f,plane} = f_f(d_{plane}) \end{cases} \quad (2)$$

where d_{mobile} and d_{plane} represent the local sliding distances of the mobile and plane surface structures, respectively.

We consider a mobile of length L moving on a plane. At a time t of the sliding number N, the mobile moved from a distance $d(t)$, as represented in Fig. 1.

At a position x of the contact, the local sliding distances are given by

$$d_{mobile}(x,t) = d(t) + D \cdot (N-1) \quad (3)$$

$$d_{plane}(x,t) = \begin{cases} (N-1) \cdot x + N \cdot d(t) & \text{if } x + d(t) < L \\ N \cdot L - x & \text{if } L < x + d(t) < D \\ N \cdot (L-x) + (N-1)(D-d) & \text{if } D < x + d(t) \end{cases} \quad (4)$$

We exemplified these evolutions in Fig. 2.

In particular, the local sliding distances are constant along the samples width, W. Thus, Eq. (1) becomes

$$F_f = W \int_0^L \left(f_{f,0}(x) + f_f(d_{mobile}(x)) + f_f(d_{plane}(x)) \right) \mathrm{d}x \quad (5)$$

The functions $f_{f,0}$ and f_f have to be identified to determine F_f. To identify the f_f function, the local sliding distances of both the mobile (d_{mobile}) and the plane (d_{plane}) have to vary separately. In the next section, we will therefore propose a method to achieve this identification. Then we will validate experimentally the model.

3. Materials and method

3.1. Methods

We call hereafter *fresh* and *old* materials that underwent no sliding and 10 repeated slidings, respectively. We carried out three

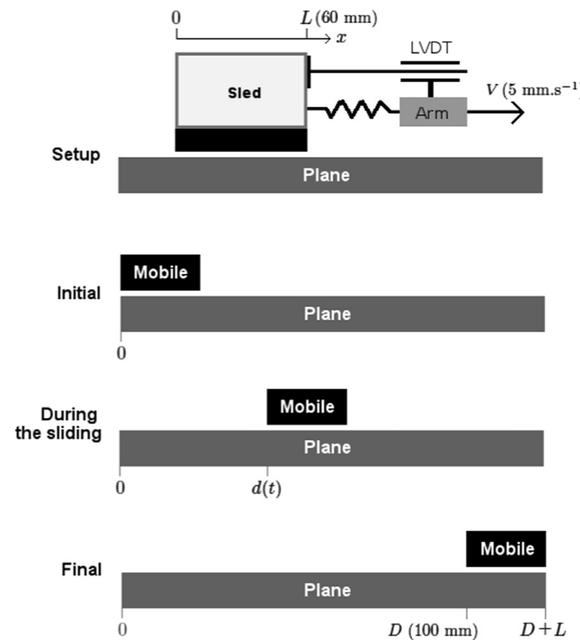

Fig. 1. Schematic view of the experimental setup. An arm moves at constant speed V. A spring and a linear variable differential transformer (LVDT) position sensor are placed between a weighted sled and the arm. The sled has a length L and slides on a plane. The rear of the mobile slides from 0 to D. At a time t during the sliding, the position of the mobile rear is $d(t)$. The position on the contact is called x.

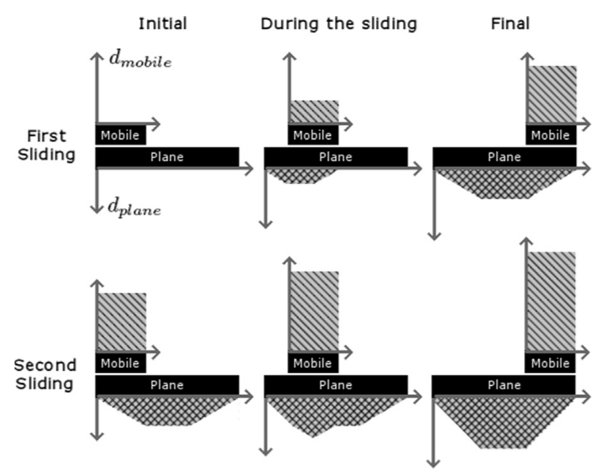

Fig. 2. Evolution of the local sliding distances of a plane (d_{plane}, double-hashed areas) and a mobile (d_{mobile}, simple hashed areas) during two repeated slidings.

different experiments involving 10 repeated slidings (from $N=1$ to 10):

(i) *PlaneChange* (PC_N) – After each repeated sliding, (i) the plane sample is replaced by a fresh one, and (ii) the mobile is lifted and placed at its initial position.
(ii) *MobileChange* (MC_N) – After each repeated sliding, the mobile sample is replaced by a fresh one, and placed at its initial position.
(iii) *NoChange* (NC_N) – After each sliding, the mobile is lifted and placed at its initial position without replacing any sample. The experiment corresponds to the standard conditions.

Each experiment is carried on 10 different pairs of samples, and the results are averaged. The local sliding distances of the materials at the beginning of the second repeated sliding of each experiment are represented in Fig. 3.

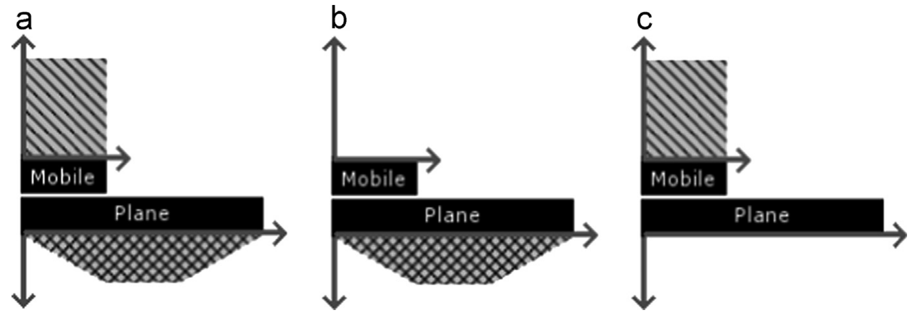

Fig. 3. Local sliding distances of the plane (d_{plane}, double-hashed areas) and the mobile (d_{mobile}, simple hashed areas) at the start of the second sliding. (a) *NoChange*. (b) *MobileChange*. (c) *PlaneChange*.

To identify the model, we measure the friction force at the beginning of each PC_N experiment, that is, when the mobile starts sliding. At the beginning of the PC_N experiments, the plane sample is fresh. Thus, the contribution of its surface structures to the friction force remains constant: $f_f(d_{plane}) = f_f(0)$. On the other hand, the local sliding distance of the mobile is homogeneous along the mobile length and increases linearly with the sliding number, N, as represented in Fig. 3(c). The contribution of the plane's surface structures to the friction force becomes $f_f(d_{mobile}) = f_f(D(N-1))$, with D the total sliding distance of the sled (see Eq. (3)). This method allows the identification of the $f_f(d)$ function, and finally the identification of the whole model.

To validate the model, we calculate the evolution of the friction force during the whole *MobileChange*, *PlaneChange*, and *NoChange* experiments. The theoretical results are compared to the results of the measurements.

3.2. Measurement method

The experiments are all carried at 23 °C and a relative humidity of 50%. The experimental protocol and the apparatus used to measure the friction force are in accordance with the standard horizontal plane test methods [10,11] (tribometer model 225-1 by the Thwing-Albert Company). The plane sample (300 mm × 210 mm) is stuck on the horizontal plane and the mobile sample on the sled. We checked that the normal load (from 600 g to 2 kg) and the velocity (from 1 mm s^{-1} to 80 mm s^{-1}) of the paper-on-paper contact have a negligible influence on the force of friction. Thus, we use a sled weighting 837 g and a contact surface of (60 mm × 60 mm). The velocity of the arm is set to 5 mm s^{-1}.

A spring is placed between the arm and the sled, as represented in Fig. 1. The spring stiffness equals 390 N m^{-1}. The spring induces a stick-slip phenomenon at roughly 2 Hz (cyclical oscillations between static and kinetic frictional states). A linear variable differential transformer (LVDT) position sensor (Sensorex 12F5 under Schaevitz license, accuracy ±0.01 mm) is placed between the sled and the arm, parallel to the spring. The frequency of acquisition is 400 Hz and the measurements are processed using a Labview program we developed. The LVDT measures the elongation of the spring. The spring stiffness being known, the LVDT thus characterizes the pulling force applied by the arm on the sled. Moreover, the second derivative of the LVDT measurement represents the sled acceleration. Thus, the friction force can be calculated by removing the mass–acceleration component from the pulling force. The method was shown to give lower dispersions and better accuracies than the standard horizontal plane test method for both the coefficients of static and kinetic friction [12]. The method also allows the measurement of the kinetics of the force of friction during the sliding, rather than just its average on the whole sliding.

To give an order of magnitude of the forces involved, we represent results as forces of friction. A calculation of the coefficients of friction is possible by dividing the force of friction by the normal load (approx. 8.2 N). Indeed, this normal load remains identical for all the experiments.

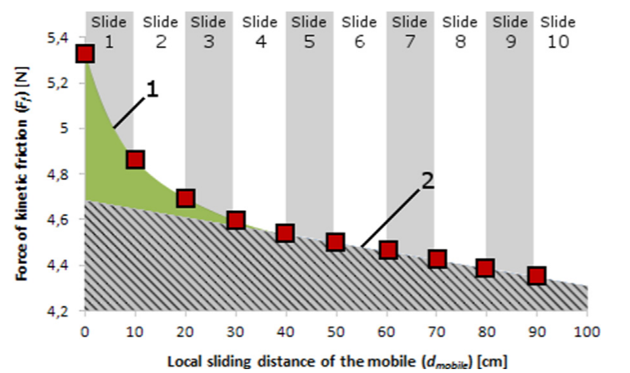

Fig. 4. Friction force at the start of each repeated sliding of the *PlaneChange* experiments (squares). The friction force is represented as a function of the local sliding distance of the mobile: for example, the start of the third repeated sliding corresponds to a local sliding distance of the mobile of 2 × 10 = 20 cm. The proposed model consists in the sum of a logarithmic-shape decrease (1) and a linear decrease (2).

3.3. Materials

We use writing papers (basis weight 80 g m^{-2}). The papers were stored 48 h at 24 °C and 50% RH. For each experiment, 10 couples of materials are tested. As paper is anisotropic, the sliding is undergone in the direction of the main fiber orientation (called machine direction). Furthermore, we place the same paper sides in contact. This can be achieved by using the same sheet of paper for the two samples.

4. Results and discussions

4.1. Identification of the model parameters

The friction forces measured at the beginnings of the PC_N experiments are represented in Fig. 4.

The force is the sum of a logarithmic-shape and a linear decreases in friction force with the local sliding distance of the mobile, similar to the observation of Broughton and Gregg with the *NoChange* experiments [5]. However, the logarithmic-shape decrease, $F_1(d_{mobile})$, appears to be better described by a sixth-order polynomial decrease:

$$F_1(d_{mobile}) = \sum_{i=0}^{6} \alpha_i d_{mobile}^i \quad (6)$$

where α_i represent the parameters of the model. The choice of a sixth-order polynomial model has no physical meaning and is only

Table 1
Parameters of the model describing the friction force evolution during the *PlaneChange* experiment: $F_f = F_1 + F_2$. Parameter estimations are based on the average results obtained on 10 different pairs of materials.

Equation	Parameter	Estimation	R^2
$F_1(d_{mobile}) = \sum_{i=0}^{6} \alpha_i d_{mobile}^i$	α_0	0.66 N	1.0
	α_1	-7.6×10^{-2} N m^{-1}	
	α_2	4.7×10^{-3} N m^{-2}	
	α_3	-1.7×10^{-4} N m^{-3}	
	α_4	3.6×10^{-6} N m^{-4}	
	α_5	-3.7×10^{-8} N m^{-5}	
	α_6	1.5×10^{-10} N m^{-6}	
$F_2(d_{mobile}) = \beta_0 + \beta_1 \cdot d_{mobile}$	β_0	4.7 N	0.98
	β_1	-3.8×10^{-3} N m^{-1}	

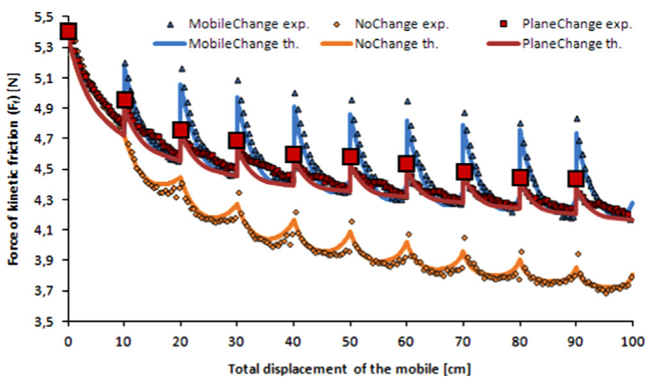

Fig. 5. Friction forces during the *MobileChange*, *PlaneChange*, and *NoChange* experiments. The measurements (dots) and calculations (lines) are represented. The big squares represent the results used to identify the model as represented in Fig. 4.

chosen for the sake of model accuracy. Numerical values for those parameters are presented in Table 1.

The logarithmic-shape decrease is observed up to the fourth repeated sliding (corresponding to $d_{mobile} = 4 \times 10 = 40$ cm). We propose to explain this decrease by the reorientation of mobile surface structures, described by the f_f function. The function f_f may be written as

$$f_f(d_{sample}(x)) = \frac{F_1(d(x))}{LW} = \frac{1}{LW} \sum_{i=0}^{6} \alpha_i d_{sample}(x)^i \tag{7}$$

where $d_{sample}(x)$ represents either $d_{mobile}(x)$, or $d_{plane}(x)$.

On the other hand, the linear decrease, $F_2(d_{mobile})$, can be described by the following expression:

$$F_2(d_{mobile}) = \beta_0 + \beta_1 \cdot d_{mobile} \tag{8}$$

where β_0 and β_1 are two parameters. Numerical values for those two parameters are presented in Table 1. The linear decrease in friction force with the mobile's local sliding distance was measured up to the 10th repeated sliding ($d_{mobile} = 100$ cm). Therefore, this decrease may represent the contributions of (i) the materials with fully reoriented surface structures, $f_{f,0}$, and (ii) the plane asperities remaining fresh, $f_f(0)$, to the force of friction. In this situation, we propose this linear decrease in contribution to the friction force to be proportional to the larger local sliding distance of both materials (d_{mobile} and d_{plane}). An expression of $f_{f,0}$ may be proposed:

$$f_{f,0}(x) = \frac{1}{LW}(\beta_0 - \alpha_0) - \frac{1}{LW}\beta_1 \cdot \max(d_{mobile}(x), d_{plane}(x)) \tag{9}$$

Finally, the expression of the friction force becomes

$$F_f = \beta_0 - \alpha_0 + \frac{1}{L}\int_0^L \beta_1 \cdot \max(d_{mobile}(x), d_{plane}(x))\, dx + \frac{1}{L}\int_0^L \sum_{i=0}^{6} \alpha_i \left[d_{mobile}(x)^i + d_{plane}(x)^i\right] dx \tag{10}$$

where $d_{mobile}(x)$ and $d_{plane}(x)$ are described by Eqs. (3) and (4), respectively.

4.2. Validation of the model

We identified the two functions of the proposed model of friction, $f_{f,0}$ and f_f, see Eq. (5). To validate the model, we calculate the friction force for the *PlaneChange*, *MobileChange*, and *NoChange* experiments. The results are represented in Fig. 5. Detailed results are proposed in supplemental information.

The model is in good agreement with the experimental results. Indeed, the coefficients of correlation, R^2, between theoretical and experimental results for the three experiments (*PlaneChange*, *MobileChange*, and *NoChange*) are higher than 0.99 in the three cases. Moreover, the model highlights several remarkable properties of the decrease in friction force with the sliding distance:

(i) The differences in friction forces between the three experiments are significant.
(ii) The decrease in friction force during the first sliding is strong (-11%) and should not be averaged, as described in standards.
(iii) The friction force measured at the end of a *NoChange* sliding is equal to the friction force at the start of the next sliding.
(iv) After several repeated slidings, the force of friction measured during a given *NoChange* sliding exhibits a minimum.

4.3. Extrapolation to other test methods

The proposed model describes the evolution of the friction force as a function of dimensional parameters. An interesting extrapolation of this model consists in comparing contacts of different natures from a contact renewal point of view. Some contacts are closed, in the sense that the same zones of the materials are constantly in contact (e.g., fretting). To exemplify this situation, we consider a ring-on-ring (or disc-on-disc) tribometer, where two ring samples are rotating on each other (see Fig. 6). The local sliding distances of both samples are the same and equal to the total displacement the samples undergo, d_{mobile}. An expression of the force of friction thus becomes

$$F_f = \beta_0 + \alpha_0 + d_{mobile}(\beta_1 + 2\alpha_1) + 2\sum_{i=2}^{6} \alpha_i d_{mobile}^i \tag{11}$$

Another contact consists in two samples sliding on each other, the size of the samples being highly different. As an example, we consider the pin-on-disk tribometer, where the size of the pin is negligible compared to the size of the disk (see Fig. 6). The local sliding distance of the disk sample is negligible compared the one of the pin sample (called d_{mobile}). In this situation, we neglect the local sliding distance of the plane. An expression of the force of friction becomes

$$F_f = \beta_0 + \alpha_0 + d_{mobile}(\beta_1 + \alpha_1) + \sum_{i=2}^{6} \alpha_i d_{mobile}^i \tag{12}$$

A qualitative comparison of the evolutions of the force of friction in the case of the ring-on-ring, pin-on-disc, and horizontal plane test methods is proposed in Fig. 6. On one hand, we observe that the evolution of the friction force with the pin-on-disc and ring-on-ring setups have the same logarithmic-shape, but the decrease is stronger with the ring-on-ring setup. This result can simply be interpreted as the aging of two surfaces with the ring-on-ring method, and a single surface with the pin-on-disc test

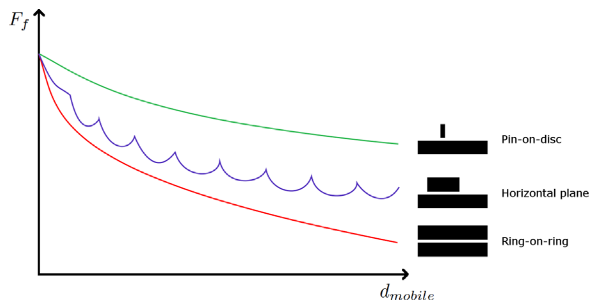

Fig. 6. Qualitative comparison of the evolution of the force of friction with the sliding distance for the pin-on-disc, the horizontal plane, and the ring-on-ring test methods. The friction force is represented as a function of the local sliding distance of the mobile sample, that is also the total displacement applied to the samples.

methods. On the other hand, the horizontal plane test method gives a more complex curve. The decrease in friction force with sliding distance remains between the pin-on-disc and ring-on-ring measurements. This behavior is explained as the sum of the aging of the mobile, plus a slower but non-negligible aging of the plane.

These results show that typical methods for friction measurement (e.g., pin-on-disc and ring-on-ring methods) give results that are qualitatively similar. Indeed, both methods give a logarithmic-shape decrease of the force of friction with the sliding distance. However, a quantitative comparison is usually considered as very difficult. This difficulty is well represented by the complex shape obtained with the horizontal plane test method (i.e., when the size of the samples are slightly different). Therefore, we hope that the proposed model could be extended to other test methods and could allow a more quantitative study of friction forces.

5. Conclusion and perspectives

- The force of friction between paper materials decreases with the sliding distance, and variations can be as high as 50%.
- We proposed a model of friction force which is able to characterize the decrease of friction with the sliding distance.
- Theoretical and experimental results are in good agreement.

The model depends on the samples sizes and the sliding distance, allowing changes in sliding scales. For example, the friction force could be calculated at a microscopic scale, allowing its comparison with microscopy measurements.

The method and model could be extended as follows:

- Complex movements at different length scales could be studied in order to extensively validate the model. In particular, we hope that the measurements obtained with different test methods could become quantitatively consistent (e.g., pin-on-disc and ring-on-ring).
- Other materials undergoing reorientations of surface structures could be studied. These reorientations may induce either decreases (e.g., textile fibers or lamellae on polymers) or increases of the force of friction (e.g., textiles with a “cat fur” effect). In particular, these changes in friction force could be non-persistent (e.g., reorientation of micro-hairs on biological surfaces).
- Finally, the method could be extended to mechanisms inducing irreversible modifications of samples surfaces, for example, wear, polishing, and abrasion.

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Appendix A. Supplementary data

Supplementary data associated with this paper can be found in the online version at http://dx.doi.org/10.1016/j.triboint.2015.07.013.

References

[1] Bureau L, Baumberger T, Caroli C. Rheological aging and rejuvenation in solid friction contacts. Eur Phys J E 2002;8:331–7. http://dx.doi.org/10.1140/epje/i2002-10017-1.
[2] Stack K. A study of friction feed paper separation. Rochester, New York: University of Rochester; 1991.
[3] Tourlonias M, Bueno MA. Etude phénoménologique du comportement au frottement de surfaces textiles. In: 19ème congrès français de mécanique, Marseille, France; 2009. p. 1–6.
[4] Vernhes P, Bloch J-F, Mercier C, Blayo A, Pineaux B. Statistical analysis of paper surface microstructure: a multi-scale approach. Appl Surf Sci 2008;254 (22):7431–7. http://dx.doi.org/10.1016/j.apsusc.2008.06.023.
[5] Broughton G, Gregg JL. Some observations on the kinetic coefficient of friction of paper. TAPPI J 1952;35:489–93.
[6] Back E. Paper-to-paper and paper-to-metal friction. In: TAPPI proceedings – international paper physics conference. Kona, Hawaii, USA; 1991. p. 49–65.
[7] Johansson DGA, Fellers C, Haugen U. Paper friction—influence of measurement conditions. TAPPI J 1988;81(5):175–83.
[8] Garoff CFN, Nilvebrant N-O. Friction hysteresis of paper. Wear 2003;256 (1):190–6. http://dx.doi.org/10.1016/S0043-1648(03)00404-6.
[9] International Standard Organization. Paper and board—determination of the static and kinetic coefficients of friction—horizontal plane method. ISO standards 15359; 1999.
[10] Technical Association of the Pulp and Paper Industry. Coefficients of static and kinetic friction of uncoated writing and printing paper by use of the horizontal plane method. TAPPI standards T 549 pm-90; 1990.
[11] Association Francaise de Normalisation. Paper and board—paper, board, corrugated board and their components—determination of the coefficient of static friction and estimation of the coefficient of dynamic friction (dynamometer method). AFNOR standards NF Q 03-082; 1993.
[12] Fulleringer N, Bloch J-F, Forced stick-slip oscillations allow the measurement of the friction: application to paper materials, Tribol Int 91, 2015, 94–98, http://dx.doi.org/10.1016/j.triboint.2015.06.021.